FÍSICA EXPERIMENTAL IV AULA 2

Análise de Fourier

SÉRIES DE FOURIER

- Joseph Fourier, paper submetido em 1807
  - Referees: Lagrange, Laplace, Malus e Legendre
  - Funções trigonométricas podem ser combinadas de tal forma a representar qualquer função matemática

$$f(x) = \frac{a_0}{2} + \sum_n (a_n \cos(nx) + b_n \sin(nx))$$

- As constantes a_n e b_n podem ser obtidas a partir de

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\cos(nx)dx$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\sin(nx)dx$$

SÉRIES DE FOURIER

- Hoje em dia, usamos formalismos mais abrangentes

$$f(x) = \sum_{n=-\infty}^{\infty} c_n e^{jnx}$$

Use a fórmula de Euler e substitua na expressão anterior

$$e^{jx} = \cos x + j \sin x$$

- Com:

$$c_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) e^{-jnx} dx$$

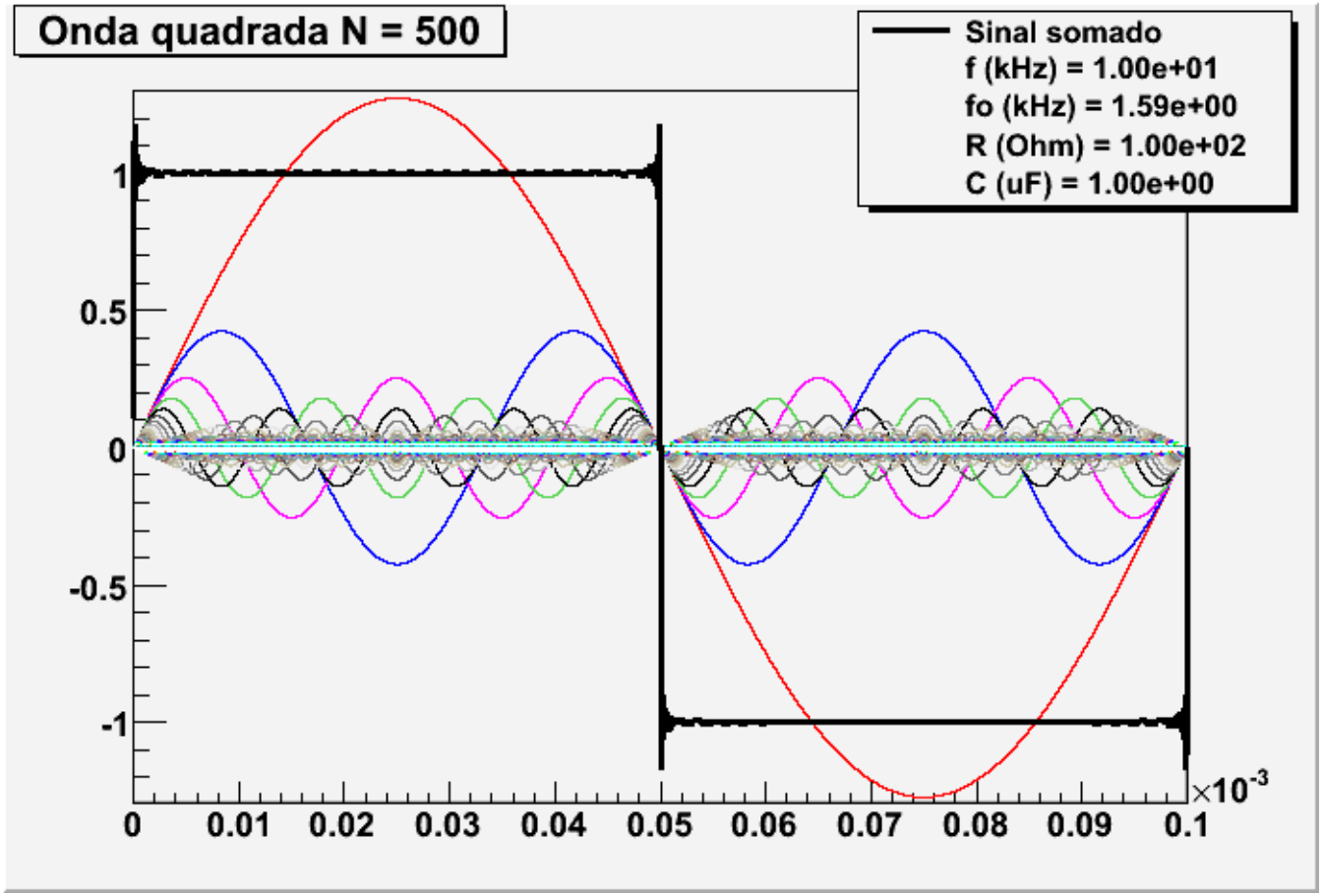
- As constantes a_n e b_n da expressão tradicional podem ser obtidas como:

$$a_n = c_n + c_{-n}, \text{ com } n = 0,1,2,\ldots$$

$$b_n = j(c_n - c_{-n}), \text{ com } n = 0,1,2,\ldots$$

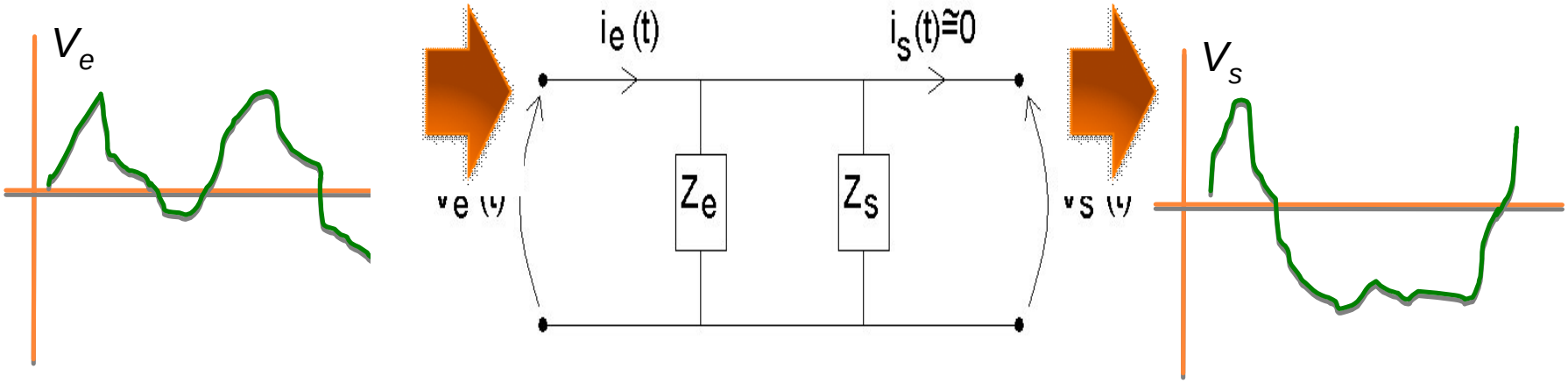
EXEMPLO: ONDA QUADRADA

$$V(t) = V_0\left[\frac{4}{\pi}\sin(\omega t) + \frac{4}{3\pi}\sin(3\omega t) + \frac{4}{5\pi}\sin(5\omega t) + \ldots\right]$$



SÉRIES DE FOURIER

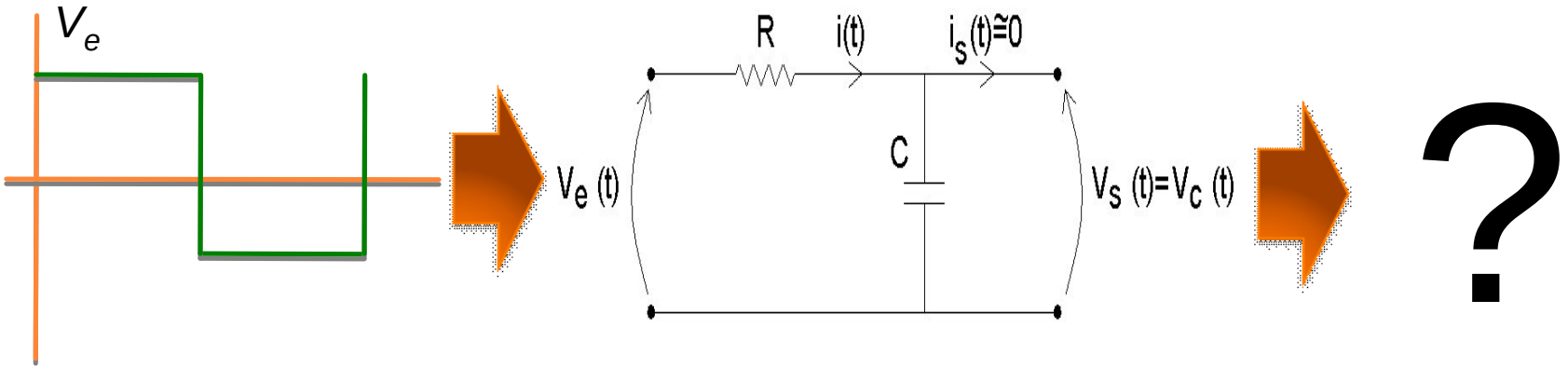
- Circuitos podem causar distorções em sinais elétricos



$$V_e = \begin{bmatrix} V_1^s \sin(\omega_1 t) + \\ V_1^c \cos(\omega_1 t) + \\ V_2^s \sin(\omega_2 t) + \\ V_2^s \cos(\omega_2 t) + \\ \ldots + \\ V_N^s \sin(\omega_N t) + \\ V_N^s \cos(\omega_N t) \end{bmatrix} \Rightarrow \begin{array}{l} G_i = G(\omega_i, R, C, \ldots) \\ \phi_i = \phi(\omega_i, R, C, \ldots) \end{array} \Rightarrow V_e = \begin{bmatrix} G_1 \cdot V_1^s \sin(\omega_1 t + \phi_1) + \\ G_1 \cdot V_1^c \cos(\omega_1 t + \phi_1) + \\ G_2 \cdot V_2^s \sin(\omega_2 t + \phi_2) + \\ G_2 \cdot V_2^s \cos(\omega_2 t + \phi_2) + \\ \ldots + \\ G_N \cdot V_N^s \sin(\omega_N t + \phi_N) + \\ G_N \cdot V_N^s \cos(\omega_N t + \phi_N) \end{bmatrix}$$

SÉRIES DE FOURIER

- O que acontece com uma onda quadrada em um filtro RC?



$$V_e = V_0\left[\frac{4}{\pi}\sin(\omega t) + \frac{4}{3\pi}\sin(3\omega t) + \frac{4}{5\pi}\sin(5\omega t) + \ldots\right]$$

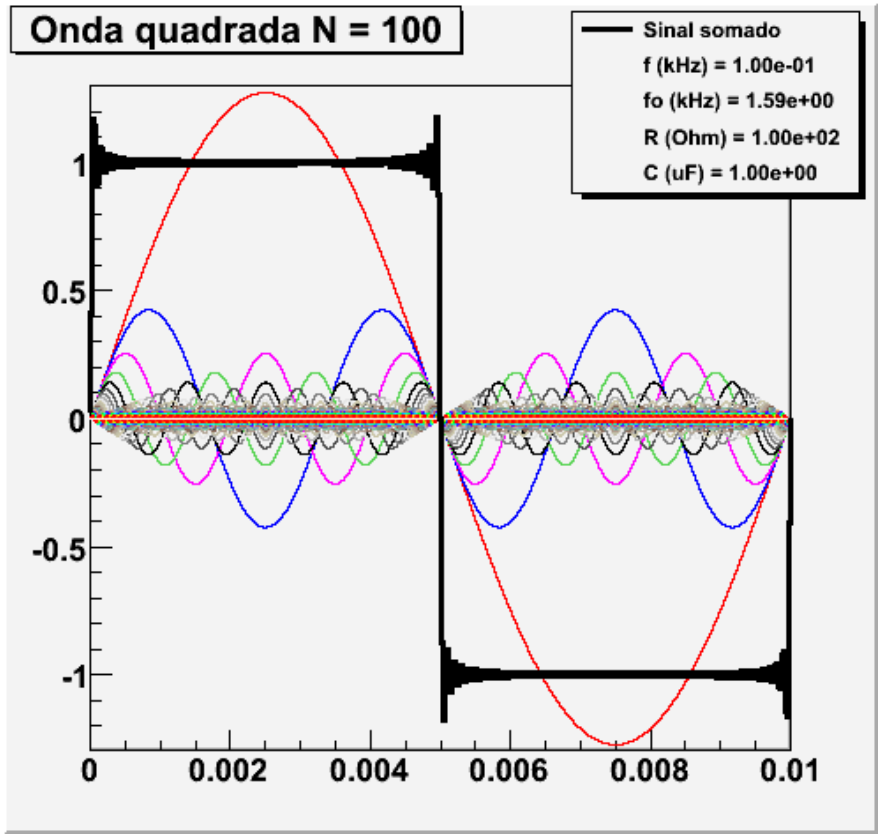
$$G(\omega) = \frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_C}\right)^2}}$$

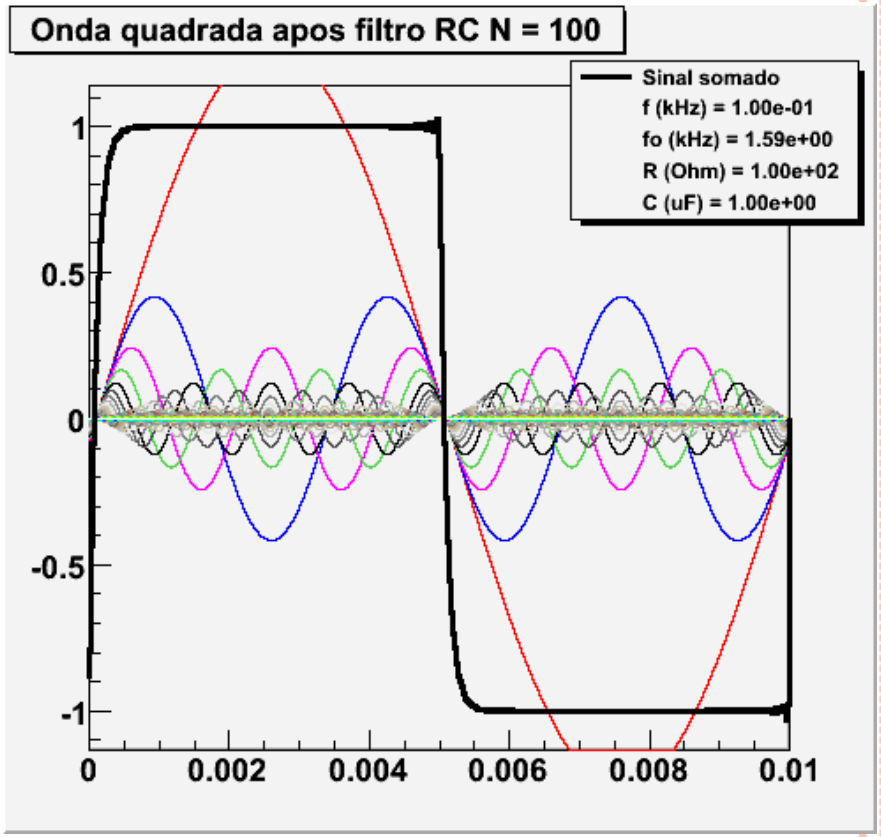
$$\phi(\omega) = \arctan\left(-\frac{\omega}{\omega_C}\right)$$

$$V_S = V_0\left[G_\omega \cdot \frac{4}{\pi}\sin(\omega t + \phi_\omega) + G_{3\omega} \cdot \frac{4}{3\pi}\sin(3\omega t + \phi_{3\omega}) + G_{5\omega} \cdot \frac{4}{5\pi}\sin(5\omega t + \phi_{5\omega}) + \ldots\right]$$

FILTRO RC (R = 1 Ω, C = 1 μF) → F_C ~ 1.5 KHZ

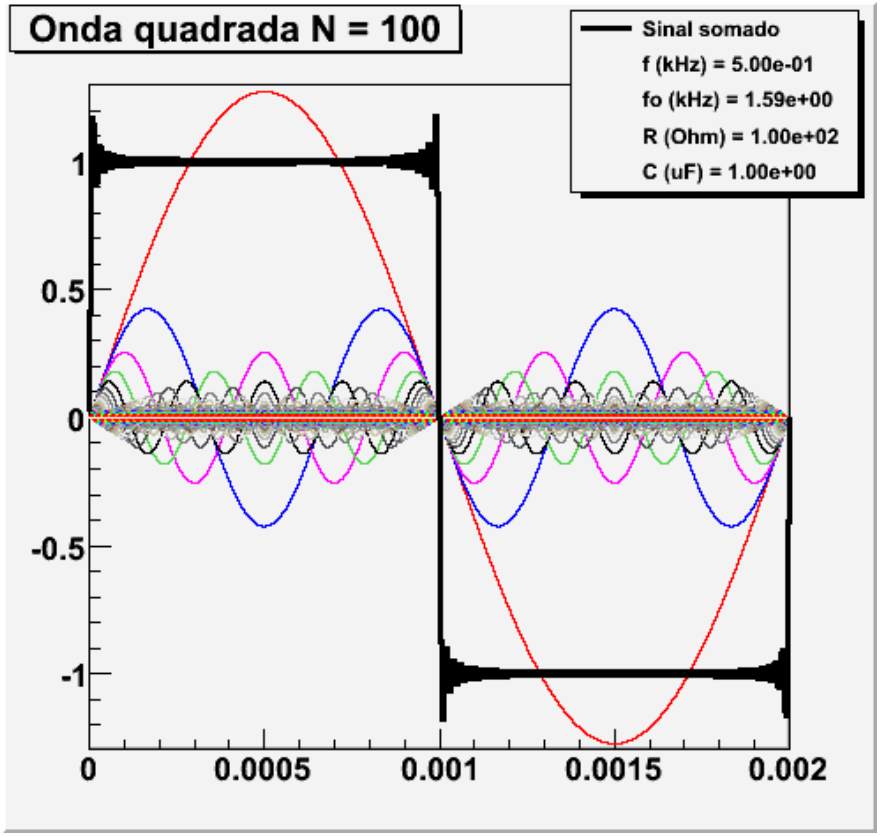
$$f_{sinal} = 100 \text{ Hz}$$

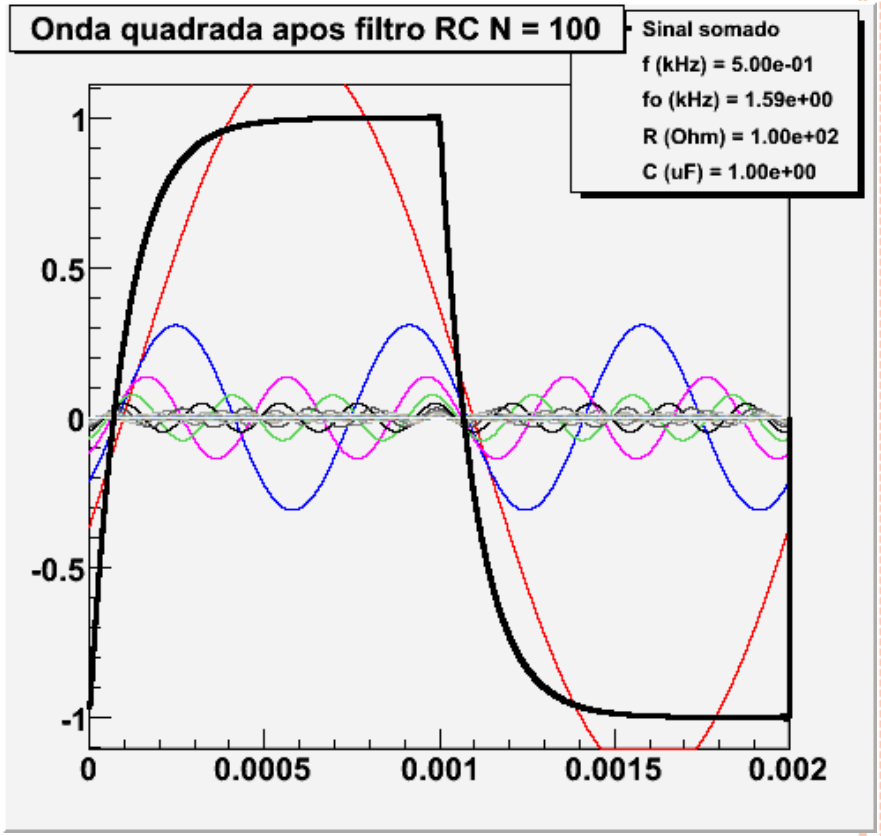




FILTRO RC (R = 1 Ω, C = 1 μF) → F_C ~ 1.5 KHZ

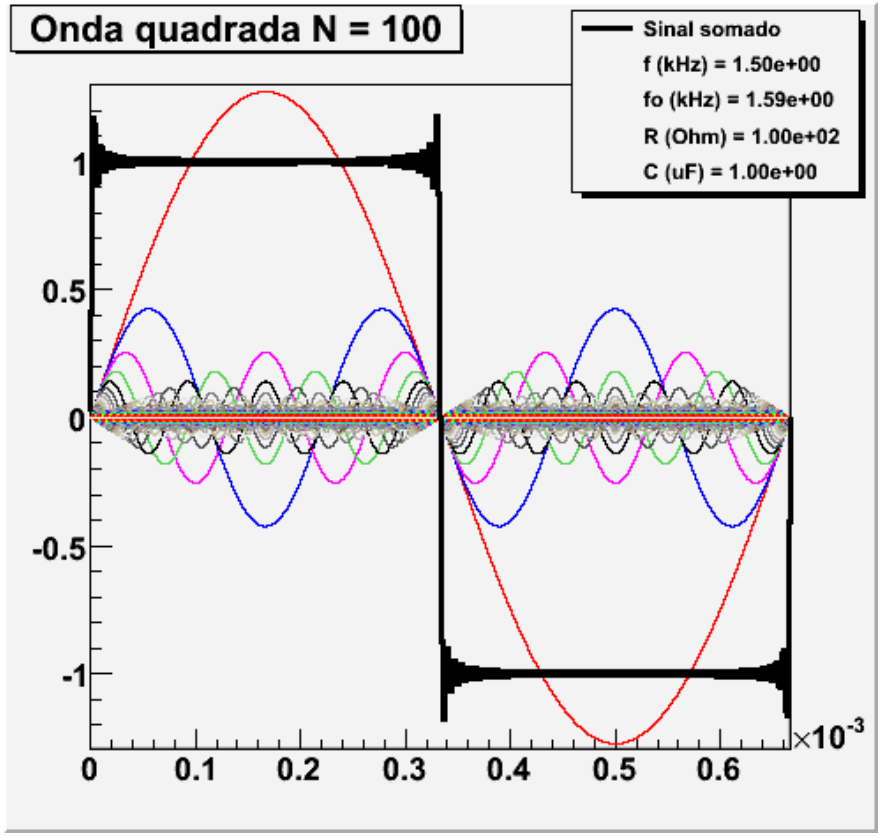
$$f_{sinal} = 500 \text{ Hz}$$

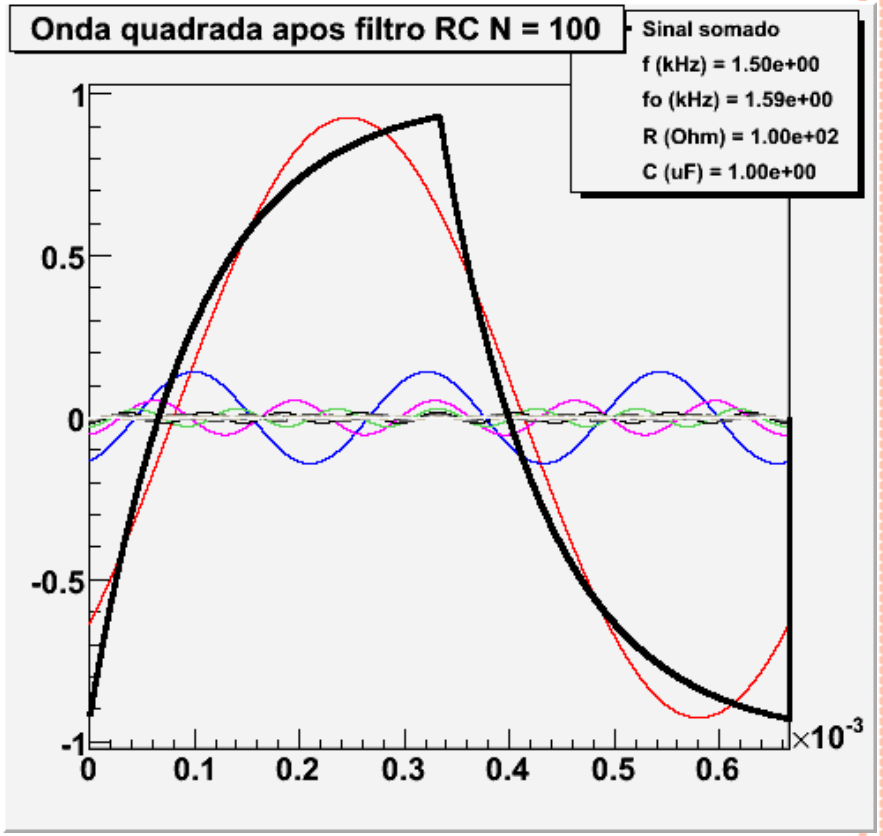




FILTRO RC (R = 1 Ω, C = 1 μF) → F_C ~ 1.5 KHZ

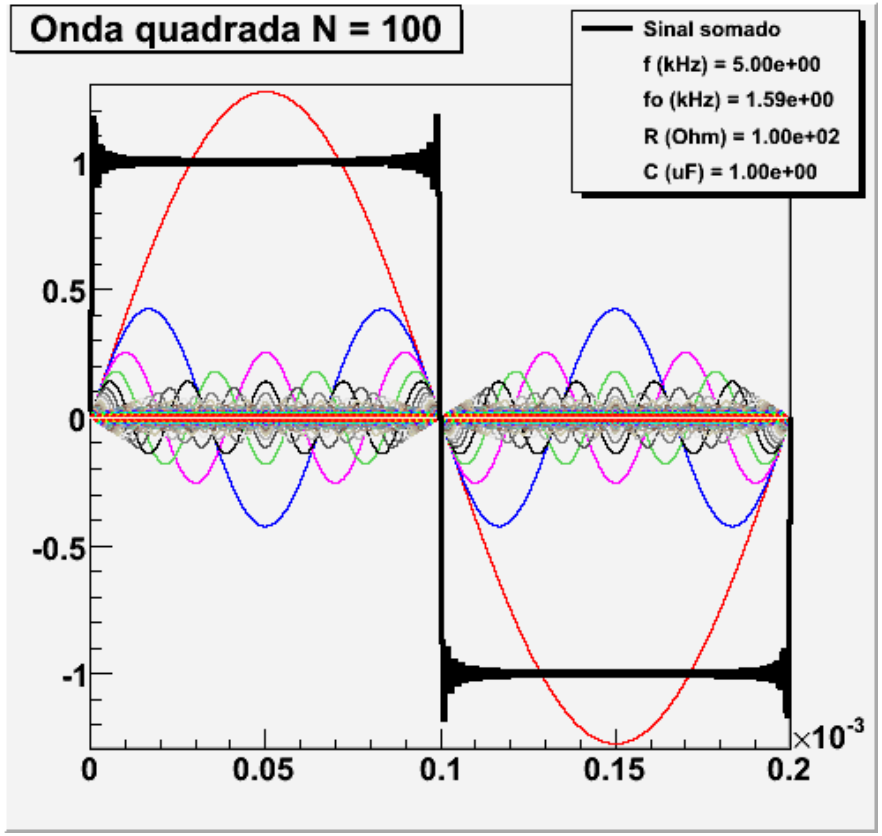
$$f_{sinal} = 1500 \text{ Hz}$$

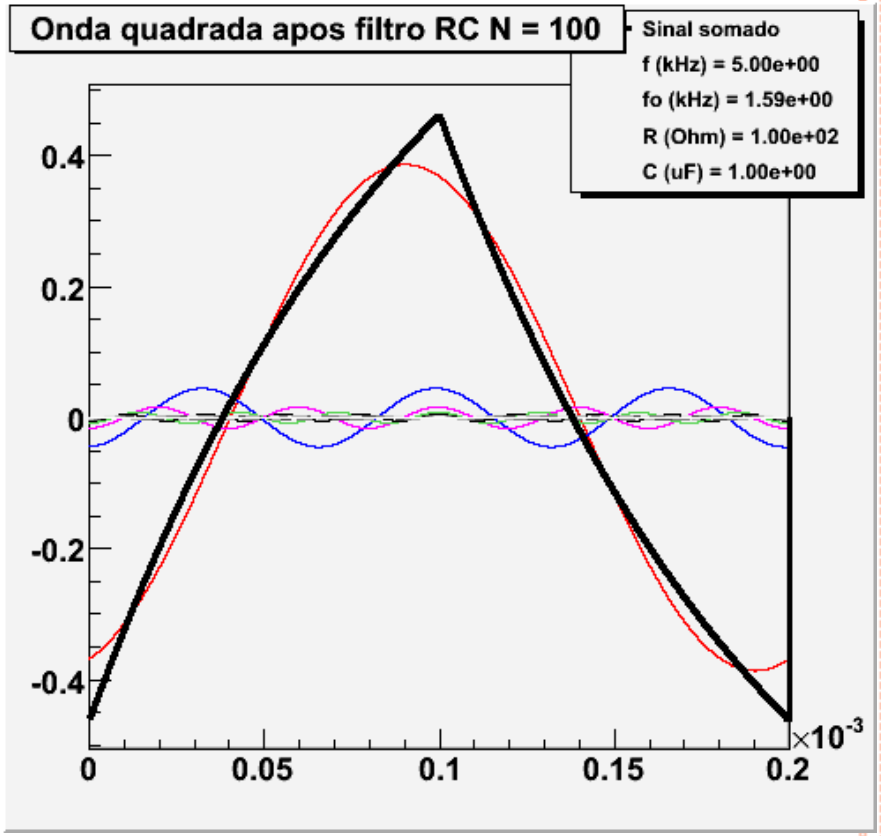




FILTRO RC (R = 1 Ω, C = 1 μF) → F_C ~ 1.5 KHZ

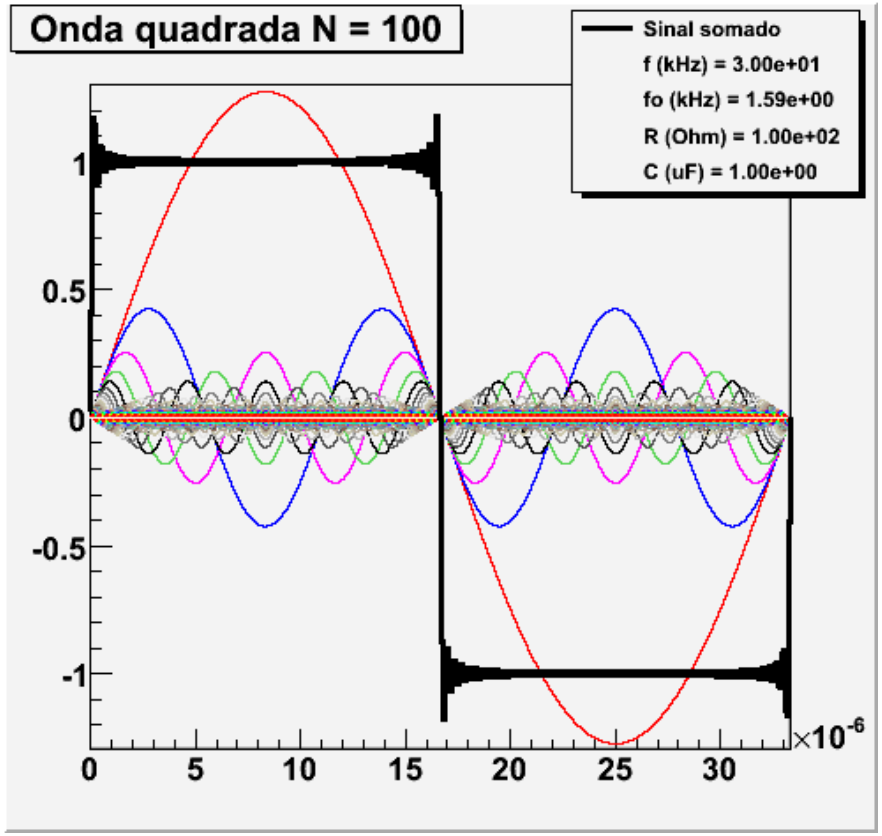
$$f_{sinal} = 5 \text{ kHz}$$

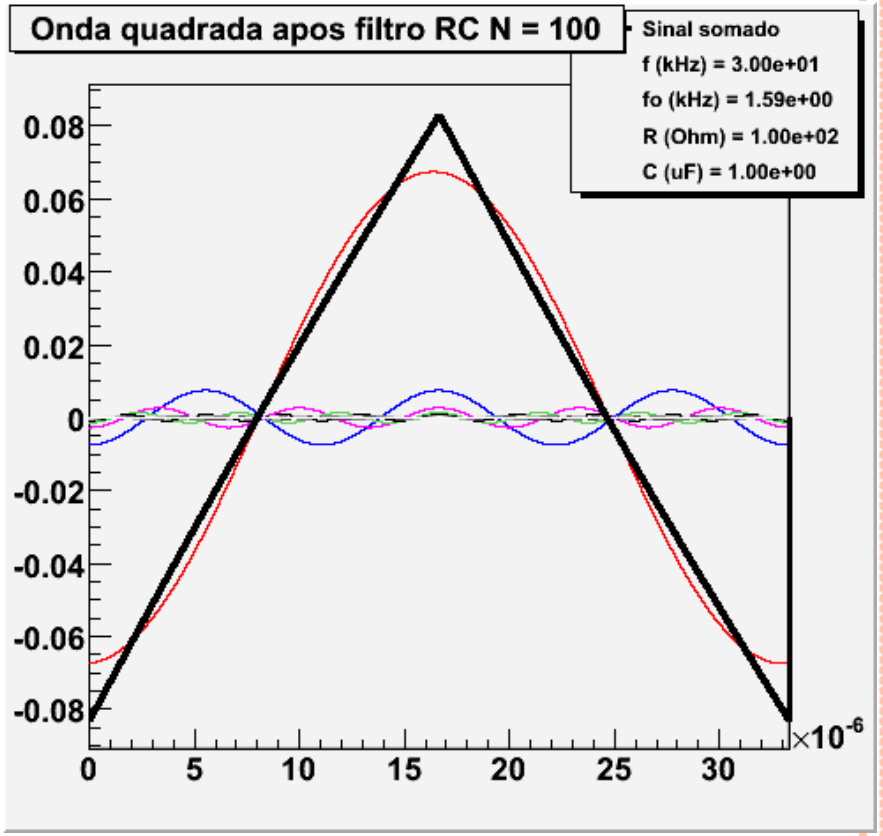




FILTRO RC (R = 1 Ω, C = 1 μF) → F_C ~ 1.5 KHZ

$$f_{sinal} = 30 \text{ kHz}$$





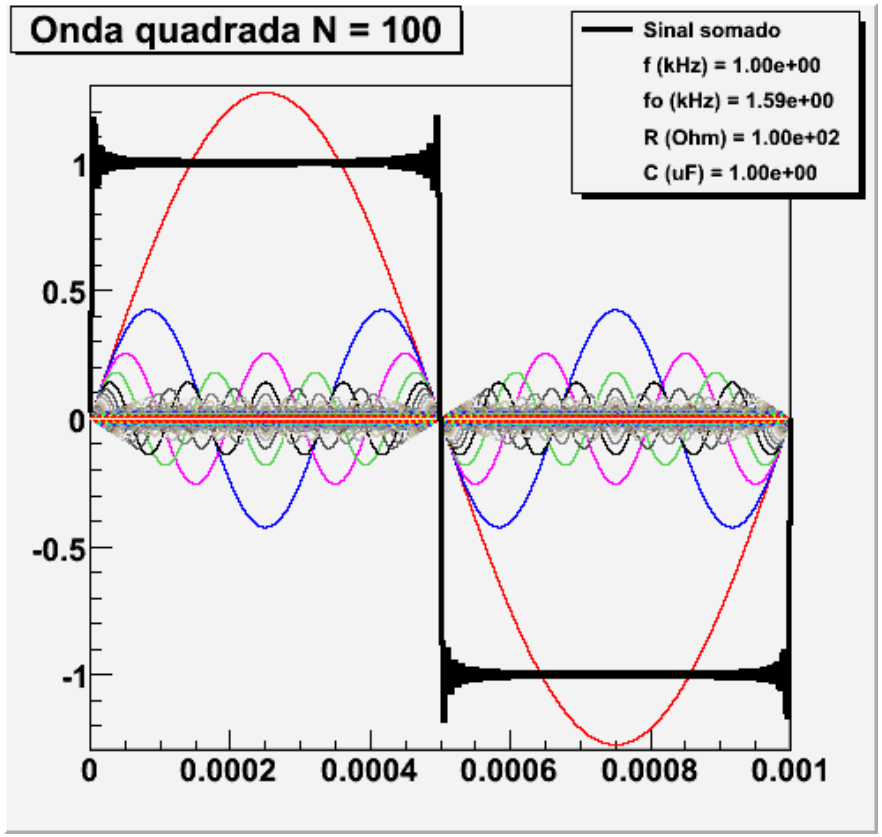
COMO ANALISAR AS FREQÜÊNCIAS DE UM SINAL

- Análise de Fourier ou transformada de Fourier
  - É um gráfico no qual o eixo-X representa a freqüência da componente de Fourier e o eixo-Y mostra a amplitude daquela componente
  - Deste modo pode-se ver claramente qual a contribuição de cada harmônica para o sinal final e podemos projetar os circuitos com o mínimo de interferência
  - Abre inúmeras possibilidades para tratamento de sinais e imagens.

- Métodos numéricos de obtenção para sinais discretos
  - FFT → Fast Fourier Transform

EXEMPLO: ONDA QUADRADA

SINAL



Transformada de Fourier
Espectro de amplitude